Exponential functions:

An example is

$$f(x) = a^x$$

where $a > 0$. What are some of the values of this function?

$$f(1) = a, \quad f(2) = a^2, \quad \text{and} \quad f(\frac{1}{2}) = \sqrt{a}.$$

There is a particularly useful example of an exponential function, namely

$$f(x) = e^x$$

where e is the base of the natural logarithm, which is

$$e = 2.71281828\ldots$$

A question you might ask is why should I care about this?

If $0 < a < 1$ then a^x appears in the mathematics behind radiometric dating and a would represent what percentage of some decaying material remains after 1 second or minute or day. One that you may have heard of is carbon-14 dating, which is used to date younger non-living organic materials. For example, if some organic material dies today and contains 2.5kg of carbon-14 (I do not know if this is a reasonable amount of carbon-14, I suspect it is not), then the formula for the amount of carbon-14 remaining after t years is

$$2.5\left(\frac{1}{2}\right)^{t/5730}.$$

So after 5730 years, there would be 1.25 kg of the carbon-14 remaining.

If $a > 1$, then a^x appears in interest computations. The number a is usually $a = (1 + \frac{r}{N})$ where r is the interest rate and N is how many times a year interest in compounded. If your savings account compounds interest monthly, your interest rate is 2% and you start with \$100 in your account then the formula for the amount of money in your savings account at time t (measured in years) is

$$100\left(1 + \frac{.02}{12}\right)^{12t}.$$

Logarithms:

If we want to undo x^2 for positive x, then we just take the square root. For example, if we want to find what number satisfies

$$x^2 = 16$$

we just take the square root of both sides, so $x = \sqrt{16} = 4$. Similarly, if we want to undo taking the square-root we square a number. So if we want to solve

$$\sqrt{x} = 4$$

then $x = (4)^2 = 16$.

If we start with a number $x > 0$, then take its square root and then square it we get back the number x. If we instead squared x first, and then take the square root we still get back x.

The logarithms do the same thing to the exponential functions. Suppose we have want to solve for b in the following equation

$$a^b = x.$$

The way we solve this is by taking the $\log_a$ of both sides:

$$a^b = x \qquad \text{and so} \qquad b = \log_a(a^b) = \log_a(x).$$

On the right-hand side, we notice that $\log_a(a^b) = b$. This is how we "undo" the exponential a^b. If $a = e$, then we write $\log_e = \ln$ and if $a = 10$ then we write $\log_{10} = \log$.

Similarly, if we want to solve for x and we have

$$\log_a(x) = b$$

then we take the exponential of both sides. That is

$$x = a^{\log_a(x)} = a^b.$$

We observe, that there was some cancellation again, that is $a^{\log_a(x)} = x$.